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A NOTE ON DEFECTLESS EXTENSIONS OF HENSELIAN VALUED FIELDS

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Abstract. A valued field (K, v) is called defectless if each of its finite extensions is defectless. In [1], Aghigh and Khanduja posed a question on defectless extensions of henselian valued fields: "if every simple algebraic extension of a henselian valued field (K, v) is defectless, then is it true that (K, v) is defectless?" They gave an example to show that the answer is "no" in general. This paper explores when the answer to the mentioned question is affirmative. More precisely, for a henselian valued field (K, v) such that each of its simple algebraic extensions is defectless, we investigate additional conditions under which (K, v) is defectless.

1. Introduction

In this paper, we consider fields equipped with (Krull) valuations. A valued field will be denoted by (K, v), its value group by vK, its residue field by Kv, and its valuation ring by $\mathcal{O}_K$. For elements $a \in K$, the value is denoted by $v(a)$. By a valued field extension $(L|K, v)$ we mean a field extension $L|K$, where v is a valuation of L and K is equipped with the restriction of v. The extension $(L|K, v)$ is called immediate if the corresponding value group and residue field extensions are trivial, i.e., if $[vL : vK] = [Lv : Kv] = 1$.

We will denote the algebraic closure of a field K by $\widetilde{K}$. By the degree of an element $\alpha \in \widetilde{K}$, we mean the degree of the extension $K(\alpha)|K$ and denote it by $\deg \alpha$. Throughout this paper, whenever we have a valuation v on a field K, we will fix an extension of v to $\widetilde{K}$, and denote it again by v. We remark that the valued field (K, v) is called henselian if the valuation v admits a unique extension to $\widetilde{K}$, or equivalently, to any algebraic extension of K. Hence for any algebraic extension $L|K$ of a henselian valued field (K, v), we consider $(L|K, v)$ to mean that L is endowed with the unique extension of v from K to L. An algebraic extension of a henselian valued field is again henselian.

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Take a finite extension $(L|K, v)$ of valued fields. It satisfies the fundamental inequality [15, Theorem 3.3.4]

$$n \geq \sum_{i=1}^{g} e_i f_i, \tag{1.1}$$

where $n = [L : K]$ is the degree of the extension, $v_1, v_2, \ldots, v_g$ are the distinct extensions of v from K to L, $e_i = [v_i L : vK]$ are the respective ramification indices and $f_i = [Lv_i : Kv]$ are the respective inertia degrees. The extension is called defectless if equality holds in (1.1). A valued field (K, v) is called defectless if each of its finite extensions is defectless, and inseparably defectless if each of its finite purely inseparable extensions is defectless.

If in addition the extension of v from K to L is unique, then the Lemma of Ostrowski (see [14, §18], [28, Ex. 32.17]) says that

$$n = p^{\nu}[vL : vK][Lv : Kv]$$

for a nonnegative integer ν and p the characteristic exponent of Kv, that is, $p = \mathrm{char} Kv$ if it is positive and $p = 1$ otherwise. The factor $d(L|K, v) := p^{\nu}$ is called the defect of the extension $(L|K, v)$. If it is nontrivial, that is, if $\nu > 0$, then $(L|K, v)$ is called a defect extension. Otherwise, as mentioned already in the general case, $(L|K, v)$ is called a defectless extension.

The notion of defectlessness plays an important role in several applications; specially, it is helpful to have equivalent characterizations because it makes the tight connection between valued fields and their invariants, value groups and residue fields. So the task of finding workable criteria for a valued field to be a defectless field is important in valuation theory. Let us first look at how the question of this note has been raised.

In case of algebraic extension of valued fields, there are some invariants associated to elements over a valued field (K, v) which play a large role in the study of extensions of valued fields and irreducible polynomials (see for example [10, 11, 17, 19]). One of the most important of such invariants is the invariant $\delta_K(\theta)$ referred to as the main invariant of an algebraic element θ over K. By the main invariant of an element $\theta \in \widetilde{K} \setminus K$ is defined the supremum (for the sake of supremum, $v\widetilde{K}$ may be viewed as a subset of its Dedekind order completion as defined in [9, Chapter III, Sec. 1, Ex. 15]) of the set $M(\theta, K)$ defined by

$$M(\theta, K) = \{v(\theta - \xi) \,|\, \xi \in \widetilde{K},\ \deg \xi < \deg \theta\}.$$

It was first defined for algebraic elements when (K, v) is a complete discrete rank one valued field [27]. Popescu and Zaharescu proved in [27] the constant $\delta_K(\theta)$ satisfies a Fundamental Principle (Theorem 2.1) which is similar to the known Krasner's Lemma [15, Theorem 4.1.7] satisfied by the Krasner constant $\omega_K(\theta)$ (see the definition in Sec. 2.1)

For a complete discrete rank one valued field (K, v), they also introduced in [27] the notions of distinguished pairs and complete distinguished chains which have recently been useful tools of valuation theory (see for example [3,4,18]). Recall that a pair (θ, α) of elements of $\widetilde{K}$ with $\deg\theta > \deg\alpha$ is called a distinguished pair (more precisely a (K, v)-distinguished pair) if α is an element of the smallest degree over K such that $v(\theta - \alpha) = \delta_K(\theta)$. Distinguished pairs give rise to distinguished chains in a natural manner. A chain $\theta = \theta_0, \theta_1, \dots, \theta_s$ of elements of $\widetilde{K}$ is called a complete distinguished chain for θ (with respect to (K, v)) if (θ_i, θ_{i+1}) is a (K, v)-distinguished pair for $0 \le i \le s-1$ and $\theta_s \in K$.

As mentioned earlier, the paper [27] contained new concepts and important results which all were satisfied for complete discrete rank one valued fields. In [1,2], Aghigh and Khanduja generalized some of its results to henselian valued fields of arbitrary rank. They utilized the concept of "defectlessness" to achieve most of their results. For example, they characterized those elements $\theta \in \widetilde{K} \setminus K$ for which there exist a distinguished pair and a complete distinguished chain by using defectless extensions of henselian valued fields as follows:

Theorem 1.1 ([1, Theorem 1.1]). *Let (K, v) be a henselian valued field, and denote again by v the unique extension of v to the algebraic closure $\widetilde{K}$ of K. Then the following two statements are equivalent*:

(i) *To each $\alpha \in \widetilde{K} \setminus K$, there corresponds $\beta \in \widetilde{K}$ with $\deg\beta < \deg\alpha$ such that $\delta_K(\alpha) = v(\alpha - \beta)$.*

(ii) *For each $\theta \in \widetilde{K}$, $(K(\theta)|K, v)$ is a defectless extension.*

Theorem 1.2 ([2, Theorem 1.2]). *Let K, v and $\widetilde{K}$ be as in the above theorem. An element $\theta \in \widetilde{K} \setminus K$ has a complete distinguished chain if and only if $(K(\theta)|K, v)$ is defectless.*

Theorems 1.1 and 1.2 shown the importance of simple algebraic extensions with the property of being defectless that in recent years have been useful tools for the study of algebraic extensions of valued fields and some properties of polynomials over valued fields (see for example [7, 8, 13, 18, 26]). In this context, a question arose about the relation between defectless simple algebraic extensions and finite defectless extensions as follows:

Question 1. For a henselian valued field (K, v), if $(K(\theta)|K, v)$ is defectless for each $\theta \in \widetilde{K} \setminus K$, then is it true that (K, v) is defectless?

An example [1, Sec. 5] was given to show that the answer to the above question is "no" in general. In this paper, it is investigated the conditions under which the answer is affirmative. In fact, we suppose that (K, v) is a henselian valued field for which every simple algebraic extension is defectless. We provide additional conditions under which every finite extension of (K, v) is defectless. It is emphasized that when K is a perfect field, the implication holds since then every finite extension is simple.

Take (K, v) to be henselian and $(K(\theta)|K, v)$ defectless for every $\theta \in \widetilde{K} \setminus K$; Theorem 3.1 shows that (K, v) is defectless whenever the Krasner constant $\omega_K(\theta)$ of θ does not exceed the main invariant $\delta_K(\theta)$ for purely inseparable elements θ over K; Theorem 3.2 proves the desired result under the additional assumption that every finite purely wild extension (see the definition in Sec. 2.2) of (K, v) is immediate; replacing this assumption by the one that every simple purely inseparable extension of (K, v) is immediate, one obtains the result of Theorem 3.3; finally, in Theorem 3.6 we show that if (K, v) is of prime characteristic and inseparably defectlee, then (K, v) is defectless.

2. Preliminaries

2.1. Invariants of algebraic elements over valued fields

Let (K, v) be any valued field and $\theta \in \widetilde{K} \setminus K$. The Krasner constant of θ over K is defined as

$$\omega_K(\theta) = \max\{v(\tau(\theta) - \sigma(\theta)) \mid \tau, \sigma \in \mathrm{Gal}(\widetilde{K}|K) \text{ and } \tau(\theta) \neq \sigma(\theta)\},$$

with the convention that $\omega_K(\theta) = \infty$ for every purely inseparable element $\theta \in \widetilde{K}$ over K. Since all extensions of v from K to $\widetilde{K}$ are conjugate, this definition does not depend on the choice of the particular extension of v. For the same reason, over a henselain field (K, v) we have that

$$\omega_K(\theta) = \max\{v(\theta - \sigma(\theta)) \mid \sigma \in \mathrm{Gal}(\widetilde{K}|K) \text{ and } \theta \neq \sigma(\theta)\}.$$

Another invariant associated with algebraic elements over valued fields is the main invariant $\delta_K(\theta)$ defined in the introduction. If (K, v) is henselian and $\theta \in \widetilde{K} \setminus K$, the main invariant of θ satisfies the Fundamental Principle as follows:

Theorem 2.1 (Fundamental Principle [20, Theorem 1.1])**.** *Let (K, v) be a henselian valued field. Let $\theta, \theta_1 \in \widetilde{K}$ be such that $v(\theta - \theta_1) > v(\theta - \gamma)$ for every $\gamma \in \widetilde{K}$ satisfying $\deg \gamma < \deg \theta$. Then*

$$vK(\theta) \subseteq vK(\theta_1), \;\; K(\theta)v \subseteq K(\theta_1)v.$$

It is noted that when every simple algebraic extension of a henselian field K is defectless, one has the result of Lemma 2.3 about the main invariant whose proof is on the same lines as that of Theorem 1.1, but for the sake of completeness, we prove it here. Before this, we need to recall the relation between immediate and defectless extensions of valued fields.

Lemma 2.2 ([21, Lemma 2.5])**.** *Take an arbitrary immediate extension $(F|K, v)$ of valued fields and a finite defectless extension $(L|K, v)$. Then the extension of v from K to $F.L$ is unique, $(F.L|F, v)$ is defectless, and $(F.L|L, v)$ is immediate. Moreover, $[F.L : F] = [L : K]$.*

Lemma 2.3. *Let (K, v) be a henselian valued field and $\theta \in \widetilde{K} \setminus K$. If $(K(\theta)|K, v)$ is defectless, then $\delta_K(\theta) \in M(\theta, K)$.*

Proof. Consider (K^c, v^c) as the completion of (K, v). Since the completion of a henselian field is an immediate extension (see [15, Theorem 1.3.4]) and from the assumption that $K(\theta)|K$ is defectless, we deduce that $[K^c(\theta) : K^c] = [K(\theta) : K]$ by Lemma 2.2. Besides, from this fact that the completion of a henselian field is again henselian (see [28, Theorem 32.19]) and by using Corollary 3.10 from [5], we obtain that $M(\theta, K)$ has an upper bound in $v\widetilde{K}$. We may consider $v\widetilde{K}$ as an ordered subgroup of its Dedekind order completion. Hence $\delta_K(\theta)$ is definable. Now assume, for the sake of obtaining a contradiction, that $\delta_K(\theta) \notin M(\theta, K)$. It means that $M(\theta, K)$ does not have a maximum element. Since $M(\theta, K)$ is a subset of totally ordered group $v\widetilde{K}$ and without last element, it contains a well-ordered cofinal subset (see [16, p. 68]). One may choose a net $\{\gamma_i\}_{i\in A}$ in $M(\theta, K)$ satisfying

(i) $\{\gamma_i\}_{i\in A}$ is cofinal in $M(\theta, K)$ and $\gamma_i < \gamma_j$ for each $i < j$, $i, j \in A$;

(ii) $\gamma_i = v(\theta - \xi_i)$, $\xi_i \in \widetilde{K}$ is such that $\deg \xi_i < \deg \theta$ and whenever $\beta \in \widetilde{K}$ has degree less than $\deg \xi_i$, then $v(\theta - \beta) < \gamma_i$.

It is noted that if necessary on replacing $\{\gamma_i\}_{i\in A}$ by a subnet, one can suppose that all ξ_i are of the same degree (say r) over K. With the assumptions above, $v(\xi_i - \xi_j) \geq \gamma_i$ and according to (ii), for every $\beta \in \widetilde{K}$ with $\deg \beta < r$, one has $v(\theta - \beta) < \gamma_i = v(\theta - \xi_i)$, hence $v(\xi_i - \beta) < v(\xi_i - \xi_j)$. This inequality together with Theorem 2.1 imply that for $i < j$, $i, j \in A$,

$$vK \subseteq vK(\xi_i) \subseteq vK(\xi_j), \tag{2.1}$$

$$Kv \subseteq K(\xi_i)v \subseteq K(\xi_j)v. \tag{2.2}$$

As all the extensions $K(\xi_i)|K$ are of the same degree $r < \deg \theta$, it is clear from (2.1) and (2.2) that there exists $j_0 \in A$ such that

$$vK(\xi_j) = vK(\xi_{j_0}),\ K(\xi_j)v = K(\xi_{j_0})v \text{ for } j \geq j_0.$$

Therefore,

$$\bigcup_{i\in A} vK(\xi_i) = vK(\xi_{j_0}),\ \bigcup_{i\in A} K(\xi_i)v = K(\xi_{j_0})v. \tag{2.3}$$

We now claim that

$$vK(\theta) = \bigcup_{i\in A} vK(\xi_i),\ K(\theta)v = \bigcup_{i\in A} K(\xi_i)v. \tag{2.4}$$

To prove the claim, let $f(x) \in K[x]$ be any polynomial of degree less than $\deg \theta$, and β be a root of $f(x)$. Since $v(\theta - \beta) \in M(\theta, K)$ and the net $\{\gamma_i\}_{i\in A}$ is cofinal in $M(\theta, K)$, there exists $l \in A$ such that $v(\theta - \beta) < \gamma_l$. Choosing l sufficiently large, we may set

$$v(\theta - \beta_k) < \gamma_l \tag{2.5}$$

for each root β_k of $f(x)$. Setting $f(x) = c\prod(x - \beta_k)$ where $c \in K$, it implies that

$$\frac{f(\theta)}{f(\xi_l)} = \prod_k \left(\frac{\theta - \beta_k}{\xi_l - \beta_k}\right) = \prod_k \left(1 + \frac{\theta - \xi_l}{\xi_l - \beta_k}\right). \tag{2.6}$$

Using the strong triangle law, the inequality of (2.5) implies that $v(\xi_l - \beta_k) = v(\theta - \beta_k)$. Consequently (2.6) shows that $v\left(\frac{f(\theta)}{f(\xi_l)} - 1\right) > 0$, namely there exists $l \in A$ such that $v(f(\theta) - f(\xi_l)) > v(f(\xi_l))$, which proves the claim.

Finally since the extension $K(\theta)|K$ is of degree $> r$, (2.3) and (2.4) immediately imply that $K(\theta)|K$ is not defectless, giving the desired contradiction. □

2.2. Ramificarion theory of valued fields

For a field K, we denote by K^{sep} the separable closure of K. If K is of characteristic $p > 0$, we denote by K^{1/p^∞} the perfect hull of K. Further, we set $K^p = \{a^p \mid a \in K\}$ and $K^{1/p} = \{a^{1/p} \mid a \in K\}$.

Let us now mention some notations and results of ramification theory (see [14, §19-22] or [24, Chapter 7]). In ramification theory, some of subgroups of $\mathrm{Gal}(\widetilde{K}|K)$ and their corresponding fixed fields are of interest. In this regard, we recall the concept of ramification fields.

Take a valued field (K, v). The fixed field of the closed subgroup

$$G^r = \{\sigma \in \mathrm{Gal}(K^{sep}|K) \mid v(\sigma(a) - a) > v(a) \text{ for all } a \in \mathcal{O}_{K^{sep}} \setminus \{0\}\}$$

of $\mathrm{Gal}(K^{sep}|K)$ is called the absolute ramification field of (K, v) and is denoted by $(K, v)^r$ or K^r if, as here, v is fixed.

Let (K, v) be henselian. The extension $K^r|K$ is defectless (see [14, §20]). For henselian valued fields Matthias Pank proved the existence of field complements of K^r in $\widetilde{K}$ which was published in [25]. Before recalling it, we need the notion of purely wild extensions. If (K, v) is a henselian valued field with $\mathrm{char}Kv = p > 0$, an algebraic extension $(L|K, v)$ is called purely wild if vL/vK is a p-group and $Lv|Kv$ is a purely inseparable extension.

Theorem 2.4 ([25]). *Let (K, v) be a henselian field with residue characteristic $p > 0$. Then*: (1) *there exist field complements L_s of K^r in K^{sep} over K, i.e., $K^r \cdot L_s = K^{sep}$ and L_s is linearly disjoint from K^r over K.* (2) *the perfect hull $L = {L_s}^{1/p^\infty}$ is a field complement of K^r over K, i.e., $K^r \cdot L = \widetilde{K}$ and L is linearly disjoint from K^r over K, and* (3) *the valued fields (L_s, v) can be characterized as the maximal separable purely wild extensions of (K, v), and the valued fields (L, v) are the maximal purely wild extensions of (K, v).*

3. Investigating defectlessness

As mentioned in the introduction, we discuss on this problem that when being defectless of every simple algebraic extension of a henselian valued field (K, v) implies that (K, v) is defectless (Question 1). We discuss some cases as follows:

Case I. If the residue field of (K, v) is of zero characteristic, then (K, v) is defectless (see [14, Corollary 20.23]).

Case II. If K is a perfect field; specially of zero characteristic, consider (K', v) as a finite extension of (K, v). Since $K'|K$ is a simple extension, the property of defectlessness of simple algebraic extensions of K implies that $(K'|K, v)$ is defectless.

Case III. In the last case, assuming the henselian valued field (K, v) has equal prime characteristic, i.e., $\text{char}K = \text{char}Kv = p > 0$, however some of the following results and proofs remain valued even in the case of zero characteristic.

In the following theorem, we impose the relation $\delta_K(\theta) \geq \omega_K(\theta)$ on the invariants of purely inseparable elements θ to obtain the desired result.

Theorem 3.1. *Suppose (K, v) is a henselian valued field for which every simple algebraic extension is defectless. If for every purely inseparable element $\theta \in \widetilde{K} \setminus K$, the Krasner constant $\omega_K(\theta)$ does not exceed its main invariant $\delta_K(\theta)$, then (K, v) is defectless.*

Proof. If $\theta \in \widetilde{K} \setminus K$ is purely inseparable over K, then $\omega_K(\theta) = \infty$ by convention. On the other hand, since $K(\theta)|K$ is defectless, the main invariant $\delta_K(\theta)$ is definable and $\delta_K(\theta) \in M(\theta, K)$ by Lemma 2.3. In fact, there corresponds $\xi \in \widetilde{K}$ with $\deg \xi < \deg \theta$ such that $v(\theta - \xi) = \delta_K(\theta)$. Therefore, $\delta_K(\theta) \geq \omega_K(\theta)$ implies $\delta_K(\theta) = \infty$. This means that $\theta = \xi$, which is impossible because $\deg \xi < \deg \theta$. So there is no element θ that generates a nontrivial purely inseparable extension of K; in other words, K is perfect. Hence by what explained in Case II, (K, v) is defectless. □

Take a henselian valued field (K, v) of residue characteristic $p > 0$. We see that every immediate algebraic extension of K is purely wild, but the converse does not hold in general. For example, if vK is p-divisible and Kv is perfect, then every purely wild extension of K is immediate (see [25, Lemma 5.2] or [23] for other characterizations of purely wild extensions). We impose this restriction on finite extensions of (K, v) and show that:

Theorem 3.2. *Let (K, v) be a henselian valued field for which every simple algebraic extension is defectless. If every finite purely wild extension of (K, v) is immediate, then (K, v) is defectless.*

Proof. Let (K', v) be any finite extension of (K, v). It should be shown that $K'|K$ is defectless. According to Theorem 2.4 (or see [24, Chap. 13, Sec. 8]), K' is contained in a composition field (say) $L'.K'^r$ where K'^r is a finite defectless extension of K and L' is a finite purely wild extension of K. By virtue of the hypothesis, the extension $L'|K$ is immediate; we show that it is trivial. Set $L' = K(\theta_1, \ldots, \theta_n)$ for some $\theta_1, \ldots, \theta_n \in \widetilde{K}$. Obviously the extension $K(\theta_1)|K$ is immediate (see [24, Lemma 8.1]) and defectless by the assumption; hence it is trivial (see [24, Corollary 11.7]). This implies that $\theta_1 \in K$. By the same argument, we have $\theta_2, \ldots, \theta_n \in K$; getting $L' = K$.

Now we have $K \subseteq K' \subseteq K'^r$. Since K'^r/K is defectless and by the fact that every subextension of a defectless extension is again defectless, one sees that $(K'|K, v)$ is a defectless extension, proving the theorem. □

One knows every purely inseparable algebraic extension of a henselian field is purely wild (see [23, Corollary 2.16]). Hence, replacing the assumption "every finite purely wild extension of (K, v) is immediate" in Theorem 3.2 by the condition "every finite (or even every simple) purely inseparable extension of (K, v) is immediate" leads to obtain the following result, which covers a wider range of fields and has a straightforward proof.

Theorem 3.3. *Suppose (K, v) is a henselian valued field such that each of its simple algebraic extensions is defectless. If every simple purely inseparable extension of (K, v) is immediate, then (K, v) is defectless.*

Proof. We want to show that K is perfect; hence we are in Case II, and (K, v) is defectless. Take a purely inseparable element θ over K. By the assumptions of the theorem, the simple extension $(K(\theta)|K, v)$ is defectless and immediate; hence it is trivial. This implies that $\theta \in K$, and so K is perfect. □

In order to give a more general result (Theorem 3.6), the previous theorem motivated us to apply the notion of inseparably defectless valued fields which are used in studying some classes of valued fields (for example, extermal fields), giving examples and making characterizations (see [6, 12, 21–24]).

We first give the following characterization of inseparably defectless fields presented in Lemma 3.1 of [21].

Lemma 3.4. *Let (K, v) be a valued field with $\mathrm{char}K = p > 0$. (K, v) is inseparably defectless if and only if $(K^{1/p^\infty}|K, v)$ is defectless, this holds if and only if $(K^{1/p}|K, v)$ is defectless, and equivalently if and only if $(K|K^p, v)$ is defectless.*

The following lemma shows that the property of being defetless is preserved under finite extensions.

Lemma 3.5 ([21, Lemma 4.15])**.** *Every finite extension of an inseparably defectless field of characteristic $p > 0$ is again an inseparably defectless field.*

Theorem 3.6. *Let (K, v) be a henselian valued field with $\mathrm{char}K = p > 0$. Assume that every simple algebraic extension of (K, v) and also $(K|K^p, v)$ are defetless. Then (K, v) is defectless.*

Proof. Take an arbitrary finite extension (K', v) of (K, v). Set F to be the relative separable closure of K in K'. Then $F|K$ is separable and $K'|F$ is purely inseparable. Since $F|K$ is simple, the extension $(F|K, v)$ is defectless by the assumption. On the other hand, by Lemmas 3.4 and 3.5, (F, v) is inseparably defectless. This implies that $(K'|F, v)$ is defectless. Now by the transitivity of defectless property (see [24, Chap. 11, Sec. 2]), the extension $(K'|K, v)$ is defectless. □

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