Commun. Korean Math. Soc. **0** (0), No. 0, pp. 1–0
https://doi.org/10.4134/CKMS.c170400
pISSN: 1225-1763 / eISSN: 2234-3024

SEMI-INVARIANT SUBMANIFOLDS OF $(LCS)_n$-MANIFOLD

Channabasappa Shanthappa Bagewadi, Channabasappa Shanthappa Bagewadi, and Mallannara Siddalingappa Siddesha

Abstract. In this paper the decomposition of basic equations of $(LCS)_n$-manifolds is carried out into horizontal and vertical projections. Further, we study the integrability of the distributions D, $D \oplus [\xi]$ and $D^{\perp}$ totally geodesic of semi-invariant submanifolds of $(LCS)_n$-manifold.

1. Introduction

The notion of semi-invariant submanifold is a generalization of invariant and anti-variant submanifolds of almost contact metric manifolds. The research work on these is carried out by (see, [1, 2, 4–7, 9–11, 14]). The authors ([1, 9, 10, 15]) have obtained the decomposition of basic equations of Kenmotsu, LP-Sasakian, (k, μ)-contact, LP-Cosymplectic manifolds into horizontal and vertical components and also they have studied the integrability of horizontal and vertical distributions. Further, analysis of totally umbilical and totally geodesics of submanifolds of (k, μ)-contact manifolds is done by the authors [1]. Now in this paper we carry out the analysis of the above authors for semi invariant submanifolds of $(LCS)_n$ manifolds. These manifolds are generalizations of LP-Sasakian manifolds and the study is followed by authors [12] to [13].

The paper is organized as follows. Section 2 consists of basic definitions of $(LCS)_n$-manifolds, invariant, anti-invariant and semi-invariant submanifold of a $(LCS)_n$-manifold. In Section 3, we obtain the decomposition of basic equations in horizontal and vertical projections. In Section 4, we study the integrability of horizontal and vertical distributions. In Section 5, we discuss totally umbilical and totally geodesic submanifolds.

2. Preliminaries

An n-dimensional Lorentzian manifold $\bar{M}$ is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, $\bar{M}$ admits a

Received September 26, 2017; Revised November 3, 2017; Accepted November 30, 2017.
2010 *Mathematics Subject Classification.* 53D15, 53C20, 53C05.
Key words and phrases. semi-invariant submanifold, $(LCS)_n$-manifold, totally umbilical and totally geodesic, integrability condition of distribution.

smooth symmetric tensor field g of type (0,2) such that for each point the tensor $g_p : T_P\bar{M} \times T_P\bar{M} \to R$ is a non-degenerate bilinear form of signature $(-, +, \dots, +)$, where $T_p\bar{M}$ denotes the tangent vector space of $\bar{M}$ at p and R is the real number space.

Definition 2.1. In a Lorentzian manifold $(\bar{M}, g)$ a vector field P defined by

$$g(X, P) = A(X),$$

for any $X \in \Gamma(T\bar{M})$, is said to be a concircular vector field if

$$(\bar{\nabla}_X A)(Y) = \alpha[g(X, Y) + \omega(X)A(Y)],$$

where α is a non- zero scalar function and ω is a closed 1-form and $\bar{\nabla}$ denotes the operator of covariant differentiation of $\bar{M}$ with respect to the Lorentzian metric g.

Let $\bar{M}$ admit a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1,$$

since ξ is a unit concircular vector field. It follows that there exists a non-zero 1-form η such that for

$$g(X, \xi) = \eta(X).$$

The equation of the following form holds:

$$(\bar{\nabla}_X \eta)(Y) = \alpha\{g(X, Y) + \eta(X)\eta(Y)\}, \alpha \neq 0$$

for all vector field X,Y, and α satisfying

$$(\bar{\nabla}_X \alpha) = (X\alpha) = d\alpha(X) = \rho\eta(X),$$

with ρ being a certain scalar function given by $\rho = -(\xi\alpha)$. Let us take $\phi X = \frac{1}{\alpha}\bar{\nabla}_X\xi$ from which it follows that ϕ is a symmetric (1,1) tensor and called the structure tensor manifold. Thus the Lorentzian manifold $\bar{M}$ together with unit timelike concircular vector field ξ, its associated 1-form η and a (1.1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly,$(LCS)_n$-manifold) [12]. Especially, if we take $\alpha = 1$, then we can obtain the LP-sasakian structure of Matsumoto [8]. In a $(LCS)_n$-manifold $(n > 2)$ the following relations hold.

$$\phi^2(X) = X + \eta(X)\xi,\ \eta(\xi) = -1,\ \phi\xi = 0,\quad \eta \cdot \phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y);\ \eta(X) = g(X, \xi). \tag{2.2}$$

The following formulas hold (see [3]):

$$(\bar{\nabla}_X \phi)Y = \alpha\{g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{2.3}$$

$$\bar{\nabla}_X \xi = \alpha\phi X, \tag{2.4}$$

$$g(X, \phi Y) = g(\phi X, Y). \tag{2.5}$$

Let M be an m-dimensional Riemannian manifold isometrically immersed in $\bar{M}$ and suppose that the structure vector field ξ of $\bar{M}$ is tangent to M. We denote by TM and $T^{\perp}M$ the tangent bundle and normal bundle to M. Then the submanifold M of $\bar{M}$ is called a semi-invariant submanifold if it is endowed with a pair of distributions $(D, D^{\perp})$ satisfying the conditions.

(1) $TM = D \oplus D^{\perp} \oplus [\xi]$,
(2) The distribution D is invariant by ϕ, that is, $\phi(D) = D$.
(3) The distribution $D^{\perp}$ is anti-invariant by ϕ, that is, $\phi(D^{\perp}) \subset T^{\perp}$M.

The distribution D (respectively $D^{\perp}$) is called the horizontal (respectively vertical) distribution. A semi-invariant submanifold M is said to be an invariant (respectively an anti-invariant) submanifold if we have $D^{\perp}_x = 0$ (respectively $D_x = 0$) for each $(x \in M)$. We say that M is a proper semi-invariant submanifold, if it is a semi-invariant submanifold, which is neither an invariant nor an anti-invariant submanifold. We denote by the same symbol g both metrics on $\bar{M}$ and M and the projection morphisms of TM to D and $D^{\perp}$ are denoted by P and Q respectively, for $X \in \Gamma(TM)$ and $N \in \Gamma(TM^{\perp})$, we have

$$X = PX + QX + \eta(X)\xi \tag{2.6}$$

$$\phi N = BN + CN, \tag{2.7}$$

where BN respectively CN denote the tangential (respectively normal) component ϕN

Definition 2.2. A submanifold M is said to be
(i) totally geodesic in $\bar{M}$ if $\sigma = 0$ or equivalently $A_N = 0$ for each $N \in T^{\perp}M$.
(ii) minimal in $\bar{M}$ if the curvature vector H satisfies $H = \frac{Tr(\sigma)}{\dim M} = 0$ and
(iii) totally umbilical if $\sigma(X, Y) = g(X, Y)H \ \forall X, Y \in TM$.

If M is a submanifold of a $(LCS)_n$-manifold $\bar{M}$ with induced metric g, then let ∇ and $\nabla^{\perp}$ be the induced connections on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M respectively. The Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{2.8}$$

$$\bar{\nabla}_X N = -A_N X + \nabla^{\perp}_X N \tag{2.9}$$

for all $X, Y \in \Gamma(TM)$ and $N \in \Gamma(T^{\perp}M)$, where h and A_N are second fundamental form and the shape operator (corresponding to the normal field N) respectively for the immersion of M into $\bar{M}$. The second fundamental form h and the shape operator A_N are related by [28]

$$g(h(X, Y), N) = g(A_N X, Y). \tag{2.10}$$

3. Decomposition of basic equations along horizontal and vertical projections;

For $X, Y \in \Gamma(TM)$, put

$$u(X, Y) = \nabla_X \phi PY - A_{\phi QY} X. \tag{3.1}$$

We begin with the following lemma:

Lemma 3.1. *Let M be a semi-invariant submanifold of $(LCS)_n$ manifold $\bar{M}$. Then we have*

$$P(u(X,Y)) = \phi P\nabla_X Y + \eta(Y)\alpha PX + 2\eta(X)\eta(Y)\alpha P\xi + g(X,Y)\alpha P\xi, \tag{3.2}$$

$$Q(u(X,Y)) = Bh(X,Y) + \eta(Y)\alpha QX + 2\eta(X)\eta(Y)\alpha Q\xi + g(X,Y)\alpha Q\xi, \tag{3.3}$$

$$h(X,\phi PY) + \nabla^{\perp}_X \phi QY = \phi Q\nabla_X Y + ch(X,Y), \tag{3.4}$$

$$\eta(u(X,Y)) = -g(\phi X, \phi Y). \tag{3.5}$$

for all $X, Y \in TM$.

Proof. We decompose (2.3) as follows Using (2.6), (2.7), in RHS of (2.3) we have.

$$\begin{aligned} \text{RHS of (2.3)} &= \alpha\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\} \\ &= g(X,Y)\alpha P\xi + g(X,Y)\alpha Q\xi + 2\eta(X)\eta(Y)\alpha P\xi \\ &\quad + 2\eta(X)\eta(Y)\alpha Q\xi + \eta(Y)\alpha PX + \eta(Y)\alpha QX + \eta(Y)\eta(X)\alpha\xi. \end{aligned} \tag{3.6}$$

Using (2.8), (2.9), in LHS of (2.3) we have

$$\begin{aligned} \text{LHS of (2.3)} &= \bar{\nabla}_X \phi Y - \phi\bar{\nabla}_X Y \\ &= \bar{\nabla}_X \phi PY + \bar{\nabla}_X \phi QY - \phi\nabla_X Y - \phi h(X,Y) \\ &= \nabla_X \phi PY + h(X,\phi PY) - A_{\phi QY}X + \nabla^{\perp}_X \phi QY \\ &\quad - \phi P\nabla_X Y - \phi Q\nabla_X Y - Bh(X,Y) - Ch(X,Y). \end{aligned}$$

Using (3.1) in the above

$$\begin{aligned} &= u(X,Y) + h(X,\phi pY) + \nabla^{\perp}_X \phi QY - \phi P\nabla_X Y \\ &\quad - \phi Q\nabla_X Y - Bh(X,Y) - Ch(X,Y) \\ &= P(u(X,Y)) + Q(u(X,Y)) + \eta_\alpha(u(X,Y))\xi_\alpha \\ &\quad + h(X,\phi pY) + \nabla^{\perp}_X \phi QY - \phi P\nabla_X Y \\ &\quad - \phi Q\nabla_X Y - Bh(X,Y) - Ch(X,Y). \end{aligned} \tag{3.7}$$

Comparing equation (3.6) and (3.7) and equating the horizontal and vertical components, we obtain (3.2), (3.3), (3.4), (3.5) respectively. □

Lemma 3.2. *Let M be a semi-invariant submanifold of $(LCS)_n$ manifold $\bar{M}$. Then we have*

$$\nabla_X \xi = \alpha\phi PX, \quad h(X,\xi) = 0 \text{ for any } X \in \Gamma(D); \tag{3.8}$$

$$\nabla_Y \xi = 0, \quad h(Y,\xi) = \alpha\phi QY \text{ for any } Y \in \Gamma(D^{\perp}); \tag{3.9}$$

$$\nabla_\xi \xi = 0, \quad h(\xi,\xi) = 0. \tag{3.10}$$

Proof. In consequence of (2.4) and (2.6), we obtain

$$\bar{\nabla}_X \xi = \nabla_X \xi + h(X,\xi).$$

Equating tangential and normal components

$$\bar{\nabla}_X \xi = \alpha\phi X,\ \ h(X,\xi) = 0$$

$$\nabla_X \xi + h(X,\xi) = \alpha(\phi PX + \phi QY). \tag{3.11}$$

Thus (3.8)-(3.10) follow from (3.11). □

Lemma 3.3. *Let M be a semi-invariant submanifold of $(LCS)_n$ manifold $\bar{M}$. Then we find*

$$\nabla_\xi U \in \Gamma(D);\ \textit{for any}\ U \in \Gamma(D),$$
$$\nabla_\xi V \in \Gamma(D^\perp);\ \textit{for any}\ V \in \Gamma(D^\perp).$$

Proof. The above follow from $g(\xi, U) = 0$, $g(\xi, V) = 0$ and (3.10) and covariant differentiation. □

Lemma 3.4. *Let M be a semi-invariant submanifold of a $(LCS)_n$ manifold $\bar{M}$. Then we obtain*

$$[X, \xi] \in \Gamma(D)\ \textit{for any}\ X \in \Gamma(D), \tag{3.12}$$

$$[Y, \xi] \in \Gamma(D^\perp)\ \ \textit{for any}\ Y \in \Gamma(D^\perp). \tag{3.13}$$

The proof follows from Lemma 3.3.

4. Integrability of invariant and anti-invariant submanifolds

In this section we study integrability of D, $D \oplus [\xi]$ and $D^\perp$ of semi-invariant submanifolds of $(LCS)_n$-manifold.

Proposition 4.1. *Let M be a semi-invariant submanifold such that ξ is tangent to $\bar{M}$. Then the invariant distribution D is integrable.*

Proof. We have for $X, Y \in D$ and $\xi \in [\xi]$

$$g([X,\, Y], \xi) = g(\nabla_X\, Y\, -\, \nabla_Y\, X,\, \xi).$$

By virtue of (2.8)

$$\begin{aligned} g([X,\, Y], \xi) &= g(\bar{\nabla}_X\, Y, + h(X, Y) - \bar{\nabla}_Y\, X, - h(Y, X), \xi) \\ &= g(\bar{\nabla}_X\, Y, \xi)\, -\, g(\bar{\nabla}_Y\, X, \xi). \end{aligned}$$

By definition of covariant differentiation

$$g([X,\, Y], \xi) = \bar{\nabla}_X g(Y, \xi) - g(Y,\, \bar{\nabla}_X \xi) - \bar{\nabla}_Y g(X, \xi) + g(X, \bar{\nabla}_Y \xi).$$

By definition of semi-invariant submanifold

$$g([X,\, Y], \xi) = -g(Y,\, \bar{\nabla}_X \xi)\, +\, g(X,\, \bar{\nabla}_Y \xi).$$

Using (2.4) we have

$$\begin{aligned} g([X,\, Y], \xi) &= -g(Y,\, \alpha\phi X)\, +\, g(X,\, \alpha\phi Y) \\ &= -\alpha[g(Y,\, \phi X)\, -\, g(X,\, \phi Y)]. \end{aligned}$$

Using (2.5) we have

$$g([X,\, Y], \xi) = 0, \tag{4.1}$$

since $\dim D \neq 0$.

Thus if $X, Y \in D$, then $[X, Y] \in D$, that is, the invariant distribution D is integrable. □

Theorem 4.2. *Let M be a semi-invariant submanifold such that each ξ of the distribution spanned by $[\xi]$ isometrically immersed in a $(LCS)_n$-manifold $\bar{M}$, then show that the distribution $D \oplus [\xi]$ is completely integrable if and only if the second fundamental form h of M satisfies*

$$h(X,\, \phi Y) = h(\phi X,\, Y) \quad \textit{for } X, Y \in D.$$

Proof. For $X, Y \in D \oplus [\xi]$ and $V \in T^{\perp}M$ then by virtue of (2.8) we have

$$\begin{aligned} g(\phi([X,\, Y]), V) &= g(\phi(\nabla_X Y \,-\, \nabla_Y X), V) \\ &= g(\phi(\bar{\nabla}_X Y \,-\, h(X,\, Y) \,-\, \bar{\nabla}_Y X \,+\, h(Y,\, X)), V) \\ &= g(\phi(\bar{\nabla}_X Y), V) - g(\phi(\bar{\nabla}_Y X, V). \end{aligned}$$

By definition of covariant differentiation

$$g(\phi([X,\, Y]), V) = g(-(\bar{\nabla}_X \phi)Y + \bar{\nabla}_X \phi Y + (\bar{\nabla}_Y \phi)X - \bar{\nabla}_Y \phi X, V).$$

Using (2.3) and (2.8) we have

$$\begin{aligned} g(\phi([X,\, Y]), V) = {} & [g(\nabla_X \phi Y, V) \,+\, g(h(X,\, \phi Y), V)) -\, g(\nabla_Y \phi X, V) \\ & - g(h(\phi X, Y), V)] + \alpha[-\, g(X,\, Y) g(\xi, Y) \\ & - 2\eta(X)\eta(Y) g(\xi, V) + \eta(Y) g(X,\, V) \,+\, g(Y,\, X) g(\xi, V) \\ & + 2\eta(Y)\eta(X) g(\xi, V) + \eta(Y) g(X,\, V)]. \end{aligned}$$

Again by virtue of (2.1) and (2.5) we have

$$g(\phi([X,\, Y]), V) = g(h(\phi X,\, Y) \,-\, h(X,\, \phi Y), V).$$

Thus $\phi([X,\, Y]) = h(\phi X,\, Y) \,-\, h(X,\, \phi Y)$, this shows that $\phi([X,\, Y])$ belongs to the orthogonal complementary distribution of $D \oplus [\xi]$ in M. Therefore $[X, Y] \in D \oplus [\xi]$ if and only if

$$h(\phi X,\, Y) \,=\, h(X,\, \phi Y).$$

□

Theorem 4.3. *Let M be a semi-invariant submanifold of a $(LCS)_n$-manifold such that ξ is tangent to $\bar{M}$ and $D^{\perp}$ be the anti-invariant subspace of TM. Then the anti-invariant distribution $D^{\perp}$ is always integrable.*

Proof. By definition of integrability we have to show that if $Z, W \in D^{\perp}$, then $[Z, W] \in D^{\perp}$ for this let $X \in TM$ then by virtue of (2.8) and by definition of covariant differentiation

$$\begin{aligned} g(\phi([Z,\, W]), X) &= g(\phi(\nabla_Z W \,-\, \nabla_W Z), X), \\ &= g(\phi(\bar{\nabla}_Z W) \,-\, \phi h(Z, W) - \phi(\bar{\nabla}_W Z) \,+\, \phi h(W, Z), X). \end{aligned}$$

Using (2.3) and (2.9) in the above we have

$$\begin{aligned}
&g(\phi([Z,\,W]),X)\\
&= g((\bar\nabla_Z\phi W)\,-\,(\bar\nabla_Z\phi)W\,-(\bar\nabla_W\phi Z)\,+\,(\bar\nabla_W\phi)Z,X)\\
&= g(A_{\phi Z}W\,-\,\nabla^{\perp}_W\phi Z+\alpha(\eta(Z)W+2\eta(Z)\eta(W)\xi+g(W,Z)\xi)\\
&\quad -A_{\phi W}Z\,+\,\nabla^{\perp}_Z\phi W-\alpha(\eta(W)Z+2\eta(Z)\eta(W)\xi+g(Z,W)\xi),X)\\
&= g(-A_{\phi W}Z+\nabla^{\perp}_Z\phi W+A_{\phi Z}W-\nabla^{\perp}_W\phi Z,X).
\end{aligned}$$

Since $A_{\phi Z}W\,-\,A_{\phi W}Z$ is tangential to M and $\nabla^{\perp}_Z\phi W\,-\,\nabla^{\perp}_W\phi Z$ is normal to M

$$g(\phi([Z,\,W]),X)=g(A_{\phi W}Z\,-\,A_{\phi Z}W),X).$$

Therefore $\phi([Z,\,W])=\ A_{\phi W}Z\,-\,A_{\phi Z}W$.

It follows that $[Z,W]\in D^{\perp}$ for any $Z,W\in D^{\perp}$ if and only if

$$A_{\phi W}Z\ =\ A_{\phi Z}W\quad\text{for any } Z,W\in D^{\perp}\tag{4.2}$$

and

$$g([Z,\,W],\,\xi)\ =\ 0\quad\text{for any } Z,W\in D^{\perp}\text{ and }\xi\in[\xi].\tag{4.3}$$

Conversely, using (2.10) and (2.8) for any $Z,W\in D^{\perp}$ and $X\in TM$ we have

$$\begin{aligned}
g(A_{\phi Z}W,\,X) &= g(h(W,\,X),\,\phi Z)\\
&= g(\bar\nabla_X W,\,\phi Z)\\
&= -g(\phi\bar\nabla_X W,\,Z)\\
&= g(-\bar\nabla_X\phi W+(\bar\nabla_X\phi)\,W,Z)\\
&= g(-\bar\nabla_X\phi W+\alpha\{g(X,W)\xi+2\eta(X)\eta(W)\xi+\eta(W)X\},Z)\\
&= g(A_{\phi W}X,\,Z)\\
&= g(A_{\phi W}Z,\,X).
\end{aligned}$$

Thus for any $Z,W\in D^{\perp}$ and $X\in TM$, $A_{\phi Z}W=A_{\phi W}Z$ holds. Hence $D^{\perp}$ is always integrable. Finally using (2.4) and taking into account that $D^{\perp}$ is an anti-invariant distribution, we have

$$\begin{aligned}
g([Z,\,W],\,\xi) &= g(\bar\nabla_Z W\,-\,\bar\nabla_W Z,\,\xi)\\
&= g(Z,\,\bar\nabla_W\xi)\,-\,g(W,\,\bar\nabla_Z\xi)\\
&= g(Z,\,\alpha\phi W)\,-\,g(W,\,\alpha\phi Z)\\
&= 0,
\end{aligned}$$

where ϕW and $\phi Z\in T^{\perp}M$ and $Z,W\in D^{\perp}\subset TM$.

Therefore (4.2) and (4.3) hold. □

5. Totally umbilical and totally geodesic submanifolds

In this section, we consider totally umbilical submanifolds of $(LCS)_n$-contact manifold. First, we prove a lemma.

Lemma 5.5. *Let D be a distribution on a submanifold M of a $(LCS)_n$-contact manifold such that $\xi \in D$. If M is D-totally umbilical, then M is D-totally geodesic.*

Proof. If M is D-totally, then by definition for all $X, Y \in D$ we have

$$h(X,Y) = g(X,Y)H,$$

where H is the mean curvature of the D-space. But in view of (3.8) and (3.10), we have

$$H = g(\xi,\xi)H = h(\xi,\xi) = 0.$$

Hence $H = 0$. Therefore M is D-totally geodesic. □

Lemma 5.6. *Let $D^\perp$ be orthogonal complementary distribution to D of a submanifold M of a $(LCS)_n$-contact manifold such that $\xi \in D^\perp$. If M is $D^\perp$-totally umbilical, then M is $D^\perp$-totally geodesic provided $\phi Q = Q\phi$.*

Proof. If M is $D^\perp$-totally, then by definition for all $X, Y \in D$ we have

$$h(X,Y) = g(X,Y)K,$$

where K is the mean curvature of the $D^\perp$-space. But in view of (3.9) we have

$$K = g(\xi,\xi)K = h(\xi,\xi) = \alpha\phi Q\xi.$$

Suppose $\phi Q = Q\phi$. Hence $K = \alpha Q\phi\xi = 0$. Therefore M is $D^\perp$-totally geodesic. □

The above Lemmas 5.5 and 5.6 imply the following two theorems:

Theorem 5.4. *Each totally umbilical submanifold M of a $(LCS)_n$-contact manifold such that ξ is tangent to M is totally geodesic.*

Theorem 5.5. *Each totally umbilical semi-invariant submanifold of a $(LCS)_n$-contact manifold is totally geodesic.*

References

[1] C. S. Bagewadi, D. Nirmala, and M. S. Siddesha, *Semi-invariant submanifolds of (k,μ)-contact manifold*, Bull. Cal. Math. Soc. **109** (2017), no. 2, 93–100.

[2] A. Bejancu, *Geometry of CR-Submanifolds*, Mathematics and its Applications (East European Series), **23**, D. Reidel Publishing Co., Dordrecht, 1986.

[3] D. E. Blair, *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, Vol. 509, Springer-Verlag, Berlin, 1976.

[4] L. S. Das, Mobin Ahmad, M. Danish Siddiqi, and A. Haseeb, *On semi-invariant submanifolds of a nearly trans-Sasakian manifold admitting a semi-symmetric semi-metric connection*, Demonstratio Math. **46** (2013), no. 2, 345–359.

[5] A. De, *A note on invariant submanifolds of (k,μ)-contact manifolds*, Ukrainian Math. J. **62** (2011), no. 11, 1803–1809.
[6] S. IanuIanuIanu and I. Mihai, *Semi-invariant submanifolds of an almost paracontact manifold*, Tensor (N.S.) **39** (1982), 195–200.
[7] M. Kobayashi, *On semi-invariant submanifolds of S-manifolds of constant f-sectional curvature*, Tensor (N.S.) **51** (1992), no. 1, 41–47.
[8] K. Matsumoto, *On Lorentzian paracontact manifolds*, Bull. Yamagata Univ. Natur. Sci. **12** (1989), no. 2, 151–156.
[9] B. Prasad, *Semi-invariant submanifolds of a Lorentzian para-Sasakian manifold*, Bull. Malaysian Math. Soc. (2) **21** (1998), no. 1, 21–26.
[10] V. S. Prasad and C. S. Bagewadi, *Semi-invariant submanifolds of Kenmotsu manifolds*, Ganita **50** (1999), no. 1, 73–81.
[11] R. Prasad, S. Sharma, and A. K. Raj, *Semi-invariant submanifolds of nearly trans-Sasakian manifolds*, Bull. of Mal. Mat. Sci. Soc. **98** (2006), no. 4, 347–366.
[12] A. Ali Shaikh, *On Lorentzian almost paracontact manifolds with a structure of the concircular type*, Kyungpook Math. J. **43** (2003), no. 2, 305–314.
[13] A. A. Shaikh and T. Q. Binh, *On weakly symmetric* $(\mathrm{LCS})_n$*-manifolds*, J. Adv. Math. Stud. **2** (2009), no. 2, 103–118.
[14] B. B. Sinha and A. K. Srivastava, *Semi-invariant submanifolds of a Kenmotsu manifold with constant ϕ-holomorphic sectional curvature*, Indian J. Pure Appl. Math. **23** (1992), no. 11, 783–789.
[15] M. M. Tripathi, *On semi-invariant submanifolds of LP-cosymplectic manifolds*, Bulletin of the Malaysian Mathematical Sciences Society **24** (2001), 69–79.

Channabasappa Shanthappa Bagewadi
Department of Mathematics
Kuvempu University
Shankaraghatta - 577 451, Shimoga, Karnataka, India
Email address: prof_bagewadi@yahoo.co.in

Channabasappa Shanthappa Bagewadi
Department of Mathematics
Kuvempu University
Shankaraghatta - 577 451, Shimoga, Karnataka, India
Email address: nirmaladraj14@gmail.com

Mallannara Siddalingappa Siddesha
Department of Mathematics
New Horizon College of Engineering
Bangalore, India
Email address: mssiddesha@gmail.com