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SOFT SOMEWHERE DENSE SETS ON SOFT TOPOLOGICAL SPACES

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Abstract. The author devotes this paper to defining a new class of generalized soft open sets, namely soft somewhere dense sets and to investigating its main features. With the help of examples, we illustrate the relationships between soft somewhere dense sets and some celebrated generalizations of soft open sets, and point out that the soft somewhere dense subsets of a soft hyperconnected space coincide with the non-null soft β-open sets. Also, we give an equivalent condition for the soft *cs*-dense sets and verify that every soft set is soft somewhere dense or soft *cs*-dense. We show that a collection of all soft somewhere dense subsets of a strongly soft hyperconnected space forms a soft filter on the universe set, and this collection with a non-null soft set form a soft topology on the universe set as well. Moreover, we derive some important results such as the property of being a soft somewhere dense set is a soft topological property and the finite product of soft somewhere dense sets is soft somewhere dense. In the end, we point out that the number of soft somewhere dense subsets of infinite soft topological space is infinite, and we present some results which associate soft somewhere dense sets with some soft topological concepts such as soft compact spaces and soft subspaces.

1. Introduction and preliminaries

In the year 1999, Molodtsov [13] proposed a completely new approach for incomplete information, namely soft sets, and he successfully applied it into several directions such as smoothness of function, Riemann integration, theory of measurement and game theory. The nature of parameters sets of soft systems provide a general framework for modeling uncertain data. This contributes towards the greatly development of theory of soft sets during a short period and gives several applications of soft sets in practical situations.

The notion of soft topological spaces was formulated by Shabir and Naz [17] in 2011. They studied the main concepts regarding soft topologies such as soft closure operators, soft subspaces and soft separation axioms. Later on, Hussain

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and Ahmad [11] continued studying the fundamental soft topological notions such as soft interior and soft boundary operators. Aygünoğlu and Aygün [7] started studying soft compactness and soft product spaces. In 2012, Zorlutuna et al. [20] introduced the first shape of soft points to study soft interior points, soft neighborhood systems and some soft maps. Then the authors of [9] and [14] did a significant contributions on soft sets and soft topologies by modifying simultaneously the notion of soft points, which play an important role to obtain many results and approach it easily. The soft filter and soft ideal notions were formulated and discussed in ([16], [18]). In 2017, the concepts of somewhere dense sets and ST_1-spaces in topological spaces were studied and investigated by me [5]. In 2018, El-Shafei et al. [10] introduced two new soft relations, namely partial belong and total non belong, and they defined partial soft separation axioms, namely p-soft T_i-spaces, for $i = 0, 1, 2, 3, 4$.

The generalizations of soft open sets play an effective role in soft topology by using them to redefine and investigate some soft concepts like soft continuous maps, soft compact spaces, soft separation axioms, etc. Most of theses generalizations are defined by utilizing soft interior and soft closure operators, and these generalizations share common properties for example a class which consists of all soft λ-open subsets of a soft topological space forms a supra soft topology on the universe set, for each $\lambda \in \{\alpha, \beta, b, \text{pre}, \text{semi}\}$. Chen [8] was the first one who studied generalized soft open sets in 2013. He defined the concepts of soft semi-open sets and soft semi-continuous maps, and studied some properties of them. At the same year, Arockiarani and Lancy [6] presented the notions of soft pre-open sets and soft regular sets. After that, Akdag and Ozkan introduced and investigated the notions of soft α-open sets [1] and soft b-open sets [2]. In 2015, Yumak and Kaymakci [19] introduced a soft β-open sets concept and studied its applications.

In this work, we initiate a soft somewhere dense sets notion as a new class of generalized soft open sets and study its main properties in detail. We show its relationships with soft regular, soft α-open, soft pre-open, soft semi-open, soft β-open and soft b-open sets, and verify that it contains all of them except for a null soft set. With regard to soft union and soft intersection, we point out under what conditions the union of soft cs-dense sets and soft intersection of soft somewhere dense sets are soft cs-dense and soft somewhere dense, respectively. Apart from that, we derive that every soft point in soft disconnected space is soft cs-dense and illustrate that a collection of soft somewhere dense subsets of a strongly soft hyperconnected space with a null soft set forms a soft topology on the universe set. Two of the significant results obtained are Theorem 2.4 which points out that every soft set is soft somewhere dense or soft cs-dense, and Theorem 2.35 which points out that the finite product of soft somewhere dense sets is soft somewhere dense. Finally, we present some results which associate soft somewhere dense sets with some soft topological concepts such as soft topological properties, soft compact spaces and soft subspaces.

In what follows, we recall some definitions and results that will be needed in the sequels.

Definition 1.1 ([13])**.** A notation G_E is said to be a soft set over X provided that G is a map of a set of parameters E into the family of all subsets of X.

Definition 1.2 ([13])**.** For a soft set G_E over X and $x \in X$, we say that:

(i) $x \in G_E$ if $x \in G(e)$ for each $e \in E$.
(ii) $x \notin G_E$ if $x \notin G(e)$ for some $e \in E$.

Definition 1.3 ([3])**.** A soft set G_E over X is called:

(i) A null soft set, denoting by $\widetilde{\emptyset}$, if $G(e) = \emptyset$ for each $e \in E$.
(ii) An absolute soft set, denoting by $\widetilde{X}$, if $G(e) = X$ for each $e \in E$.

Definition 1.4 ([3])**.** The relative complement of a soft set G_E is denoted by G^c_E, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$ for each $e \in E$.

Definition 1.5 ([3])**.** The union of two soft sets G_A and F_B over X is the soft set V_D, where $D = A \bigcup B$ and a map $V : D \to 2^X$ is defined as follows:

$$V(d) = \begin{cases} G(d) & : \quad d \in A - B \\ F(d) & : \quad d \in B - A \\ G(d) \bigcup F(d) & : \quad d \in A \bigcap B \end{cases}$$

It is written briefly, $G_A \widetilde{\bigcup} F_B = V_D$.

Definition 1.6 ([15])**.** The intersection of two soft sets G_A and F_B over X is the soft set V_D, where $D = A \bigcap B$, and a map $V : D \to 2^X$ is defined by $V(d) = G(d) \bigcap F(d)$ for all $d \in D$. It is written briefly, $G_A \widetilde{\bigcap} F_B = V_D$.

Definition 1.7 ([15])**.** A soft set G_A is a soft subset of a soft set F_B if $A \subseteq B$ and $G(a) \subseteq F(a)$ for all $a \in A$.

Definition 1.8 ([17])**.** A collection θ of soft sets over X under a parameters set E is said to be a soft topology on X if it satisfies the following three axioms:

(i) $\widetilde{X}$ and $\widetilde{\emptyset}$ belong to θ.
(ii) The intersection of a finite family of soft sets in θ belongs to θ.
(iii) The union of an arbitrary family of soft sets in θ belongs to θ.

The triple (X, θ, E) is called a soft topological space. Every member of θ is called a soft open set and its relative complement is called a soft closed set.

Proposition 1.9 ([17])**.** *If (X, θ, E) is a soft topological space, then a family $\theta_e = \{G(e) : G_E \in \theta\}$ forms a topology on X, for each $e \in E$.*

Definition 1.10 ([17])**.** Let F_E be a soft subset of (X, θ, E). Then:

(i) $cl(F_E)$ is the intersection of all soft closed subsets of (X, θ, E) containing F_E.

(ii) $(cl(F))_E$ is defined as $(cl(F))(e) = cl(F(e))$ for each $e \in E$, where the closure of $F(e)$ is taken in (X, θ_e) which defined in the above proposition.

Proposition 1.11 ([17]). *Let F_E be a soft subset of (X, θ, E). Then*

$$(cl(F))_E \widetilde{\subseteq} cl(F_E).$$

Definition 1.12 ([14]). Let Y_E be a non-null soft subset of (X, θ, E). Then $\theta_Y = \{Y_E \widetilde{\bigcap} G_E : G_E \in \theta\}$ is called a soft relative topology on Y_E and (Y_E, θ_{Y_E}, E) is said to be a soft subspace of (X, θ, E).

Definition 1.13 ([11]). Let F_E be a soft subset of (X, θ, E). Then:

(i) $int(F_E)$ is the union of all soft open subsets of (X, θ, E) contained in F_E.

(ii) A soft boundary of a soft set F_E, denoted by $b(F_E)$, is given by $b(F_E) = cl(F_E) \setminus int(F_E)$.

Theorem 1.14 ([11]). *For any soft subset B_E of (X, θ, E), we have the following results*:

(i) $int(B_E^c) = (cl(B_E))^c$.

(ii) $cl(B_E^c) = (int(B_E))^c$.

(ii) *If F_E is soft closed, then $b(F_E) \widetilde{\subseteq} F_E$.*

Definition 1.15 ([7]). Let G_A and H_B be soft sets over X and Y, respectively. Then the cartesian product of G_A and H_B is denoted by $(G \times H)_{A \times B}$ and is defined as $(G \times H)(a, b) = G(a) \times H(b)$ for each $a \in A$ and $b \in B$.

Theorem 1.16 ([7]). *Let (X_1, θ_1, E_1) and (X_2, θ_2, E_2) be soft topological spaces. A soft topology T for $X_1 \times X_2$ whose soft base is the collection $B = \{G_{E_1} \times G_{E_2} : G_{E_1} \in \theta_1$ and $G_{E_2} \in \theta_2\}$ is called product soft topology and $(X_1 \times X_2, T, E)$ is called soft product space of (X_1, θ_1, E_1) and (X_2, θ_2, E_2), where $E = E_1 \times E_2$.*

Definition 1.17 ([9]). A soft set H_E over X is called finite (resp. countable) if $H(e)$ is finite (resp. countable) for each $e \in E$.

Definition 1.18 ([9], [14]). A soft subset P_E of $\widetilde{X}$ is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(\alpha) = \emptyset$ for each $\alpha \in E \setminus \{e\}$. A soft point will be shortly denoted by P_e^x and we say that $P_e^x \in G_E$, if $x \in G(e)$.

Definition 1.19 ([14], [20]). A soft map $f_\phi : (X, \theta, A) \to (Y, \tau, B)$ is said to be:

(i) Soft continuous if the inverse image of each soft open subset of (Y, τ, B) is a soft open subset of (X, θ, A).

(ii) Soft open (resp. soft closed) if the image of each soft open (resp. soft closed) subset of (X, θ, A) is a soft open (resp. soft closed) subset of (Y, τ, B).

(iii) Soft homeomorphism if it is bijective, soft continuous and soft open.

Definition 1.20. A soft subset A_E of (X, θ, E) is said to be:

(i) Soft semi open [8] if $A_E \widetilde{\subseteq} cl(int(A_E))$.
(ii) Soft β-open [19] if $A_E \widetilde{\subseteq} cl(int(cl(A_E)))$.
(iii) Soft regular open [6] if $A_E = int(cl(A_E))$.
(iv) Soft pre open [6] if $A_E \widetilde{\subseteq} int(cl(A_E))$.
(v) Soft α-open [1] if $A_E \widetilde{\subseteq} int(cl(int(A_E)))$.
(vi) Soft b-open [2] if $A_E \widetilde{\subseteq} int(cl(A_E)) \widetilde{\bigcup} cl(int(A_E))$.

Definition 1.21 ([5])**.** A subset A of a topological space (X, θ) is called somewhere dense if $int(cl(A)) \neq \emptyset$.

Definition 1.22 ([16])**.** A non-null soft collection $\mathcal{F}$ of soft subsets of $\widetilde{X}$ is said to be a soft filter if it meets the next three axioms:

(i) $\widetilde{\emptyset} \notin \mathcal{F}$.
(ii) The soft superset of any member of $\mathcal{F}$ belongs to $\mathcal{F}$.
(iii) The soft intersection of any two member of $\mathcal{F}$ belongs to $\mathcal{F}$.

Definition 1.23 ([18])**.** A non-null soft collection $\mathcal{I}$ of soft subsets of $\widetilde{X}$ is said to be a solft ideal if it meets the following three axioms:

(i) $\widetilde{X} \notin \mathcal{I}$.
(ii) The soft subset of any member of $\mathcal{I}$ belongs to $\mathcal{I}$.
(iii) The soft union of any two member of $\mathcal{I}$ belongs to $\mathcal{I}$.

Definition 1.24 ([12])**.** A soft topological space (X, θ, E) with no mutually disjoint non-null soft open sets is called soft hyperconnected.

2. Soft somewhere dense sets

Definition 2.1. A soft subset D_E of (X, θ, E) is said to be soft somewhere dense if there exists a non-null soft open set G_E such that $G_E \widetilde{\subseteq} cl(D_E)$. The complement of a soft somewhere dense set is said to be soft cs-dense.

Remark 2.2. A collection of all soft somewhere dense subsets of (X, θ, E) is denoted by $SS(\theta)$.

Proposition 2.3. *If (X, θ, E) is indiscrete, then $SS(\theta) \bigcup \{\widetilde{\emptyset}\}$ is the discrete soft topology on X.*

Proof. For any non-null soft subset A_E of an indiscrete soft topological space (X, θ, E), we have that $cl(A_E) = \widetilde{X}$. Then every non-null soft subset of $\widetilde{X}$ is soft somewhere dense. This completes the proof. □

Theorem 2.4. *Every soft subset of (X, θ, E) is soft somewhere dense or soft cs-dense.*

Proof. Suppose that A_E is a soft subset of $\widetilde{X}$ that is not soft somewhere dense. Then $cl(A_E)$ has null soft interior, so we cannot have $cl(A_E) = \widetilde{X}$. Then $(cl(A_E))^c$ is a non-null soft open subset of A_E^c, hence of $cl(A_E^c)$, so A_E^c is soft somewhere dense and A_E is soft cs-dense. □

Theorem 2.5. *A soft subset B_E of (X, θ, E) is soft cs-dense if and only if there exists a proper soft closed subset F_E of $\widetilde{X}$ such that $int(B_E) \widetilde{\subseteq} F_E$.*

Proof. Suppose that B_E is a soft *cs*-dense set. Then B_E^c is soft somewhere dense. So there exists a non-null soft open set G_E satisfies that $G_E \widetilde{\subseteq} cl(B_E^c)$. Now, we can observe that $(cl(B_E^c))^c \widetilde{\subseteq} G_E^c$ and $G_E^c \neq \widetilde{X}$. It follows, by Theorem 1.14, that $int(B_E) \widetilde{\subseteq} G_E^c$. Putting $F_E = G_E^c$, hence the proof of the necessary part is made.

Conversely, consider $B_E \widetilde{\subseteq} \widetilde{X}$ and there exists a proper soft closed set F_E such that $int(B_E) \widetilde{\subseteq} F_E$. Then $F_E^c \widetilde{\subseteq} (int(B_E))^c$ and $F_E^c \neq \widetilde{\emptyset}$. Since $(int(B_E))^c = cl(B_E^c)$, then B_E^c is soft somewhere dense. Hence B_E is soft *cs*-dense. □

Corollary 2.6. *Every soft subset of a soft cs-dense set is soft cs-dense.*

Corollary 2.7. *If (X, θ, E) is soft disconnected, then every soft point in $\widetilde{X}$ is soft cs-dense.*

Proof. Suppose that $\widetilde{X}$ is soft disconnected. Then there exists a proper non-null soft subset H_E of $\widetilde{X}$ which is both soft open and soft closed. This means that there exist proper soft closed subsets F_E and L_E of $\widetilde{X}$ such that $int(H_E) \widetilde{\subseteq} F_E$ and $int(H_E^c) \widetilde{\subseteq} L_E$. So for each $P_e^x \in \widetilde{X}$, we have either $P_e^x \in H_E$ or $P_e^x \in H_E^c$. Hence we find that $int(P_e^x) \widetilde{\subseteq} F_E$ or $int(P_e^x) \widetilde{\subseteq} L_E$. This completes the proof. □

Proposition 2.8. *Every non-null soft β-open set is soft somewhere dense.*

Proof. Assume that D_E is a non-null soft β-open set. Then

$$D_E \widetilde{\subseteq} cl(int(cl(D_E))) \widetilde{\subseteq} cl(cl(D_E)) = cl(D_E).$$

Therefore $int(cl(D_E))$ is a non-null soft open set contained in $cl(D_E)$. This automatically means that D_E is a soft somewhere dense set. □

The example below shows that the converse of the above proposition fails.

Example 2.9. Let $E = \{e_1, e_2\}$ be a set of parameters and the four soft sets G_{1_E}, G_{2_E}, G_{3_E} and G_{4_E} over $X = \{x, y, w, z\}$ be defined as follows:

$$G_1(e_1) = \{x\},\ G_1(e_2) = \emptyset;$$
$$G_2(e_1) = \{x, y\},\ G_2(e_2) = \{w, z\};$$
$$G_3(e_1) = \{w, z\},\ G_3(e_2) = \{x, y\};\ \text{and}$$
$$G_4(e_1) = \{x, w, z\},\ G_4(e_2) = \{x, y\}.$$

Then a collection $\theta = \{\widetilde{\emptyset}, \widetilde{X}, G_{1_E}, G_{2_E}, G_{3_E}, G_{4_E}\}$ forms a soft topology on X. Let F_E be a soft subset of $\widetilde{X}$ such that $F(e_1) = \{y, w, z\}$ and $F(e_2) = X$. Since $G_{3_E} \widetilde{\subseteq} cl(F_E) = F_E$, then F_E is soft somewhere dense. On the other hand, $cl(int(cl(F_E))) = G_{3_E}$. This implies that F_E is not soft β-open.

Remark 2.10. In [2], the authors showed that every soft regular, soft open, soft α-open, soft semi-open, soft pre-open and soft b-open sets are soft β-open set. Then, by Proposition 2.8, we can deduce that they are a soft somewhere dense set.

The relationships which discussed in the previous proposition and example are illustrated in the following figure.

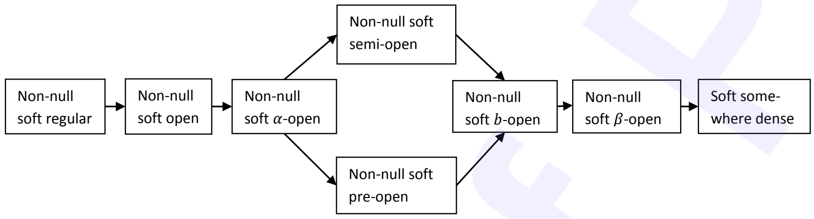

FIGURE 1. The relationships between soft somewhere dense set and some generalized soft open sets

For the sake of brevity, the proof of the following two propositions has been omitted.

Proposition 2.11. *Every non-null proper soft regular open subset of* (X, θ, E) *is both soft somewhere dense and soft cs-dense.*

Proposition 2.12. *If* (X, θ, E) *is soft hyperconnected, then a collection of soft somewhere dense sets coincides with a collection of non-null soft* β*-open sets.*

Definition 2.13. A soft subset W_E of (X, θ, E) is said to be soft S-neighborhood of $P_e^a \widetilde{\in} \widetilde{X}$ if there exists a soft somewhere dense set D_E such that $P_e^a \widetilde{\in} D_E \widetilde{\subseteq} W_E$.

Now, it is easy to prove the following three propositions.

Proposition 2.14. *A soft subset* D_E *of* (X, θ, E) *is soft somewhere dense if and only if it is soft* S*-neighbourhood of at least one soft point* $P_e^x \in D_E$.

Proposition 2.15. *Every soft neighbourhood of any soft point in* (X, θ, E) *is a soft somewhere dense set.*

Proposition 2.16. *Every proper superset of a soft somewhere dense set* D_E *is soft somewhere dense.*

Corollary 2.17. *If a soft boundary of a soft closed set F_E is soft somewhere dense, then F_E is soft somewhere dense.*

Proof. Suppose that F_E is soft closed such that $b(F_E)$ is soft somewhere dense. Then $b(F_E) \widetilde{\subseteq} F_E$. So F_E is soft somewhere dense. □

The converse Proposition 2.15 and Proposition 2.16 need not be true in general as shown in the following example.

Example 2.18. Assume that (X, θ, E) is the same as in Example 2.9. Let the two soft subsets F_E and L_E of $\widetilde{X}$ be defined as follows:

$$F(e_1) = \{y\},\ F(e_2) = \{x, w\} \text{ and}$$
$$L(e_1) = \{y\},\ L(e_2) = \{w, z\}.$$

Then we note the following:

(i) F_E is soft somewhere dense but it is not soft neighbourhood of any soft point belongs to $\widetilde{X}$.

(ii) For any proper superset H_E of L_E, we can see that H_E is soft somewhere dense, whereas L_E is not soft somewhere dense.

Theorem 2.19. *The union of an arbitrary non-empty collection of soft somewhere dense subsets of (X, θ, E) is soft somewhere dense.*

Proof. Let $\{D_{i_E} : i \in I \neq \emptyset\}$ be a family of soft somewhere dense sets. Then there exists $i_0 \in I$ and there exists a non-null soft open set G_E such that $G_E \widetilde{\subseteq} cl(D_{{i_0}_E}) \widetilde{\subseteq} cl\ (\widetilde{\bigcup}_{i\in I} D_{i_E})$. Hence $\widetilde{\bigcup}_{i\in I} D_{i_E}$ is a soft somewhere dense set. □

Corollary 2.20. *The intersection of an arbitrary non-empty collection of soft cs-dense subsets of (X, θ, E) is soft cs-dense.*

Remark 2.21. The intersection of a finite soft somewhere dense sets is not always soft somewhere dense and the union of a finite soft *cs*-dense sets is not always soft *cs*-dense as illustrated in the following example.

Example 2.22. Assume that (X, θ, E) is the same as in Example 2.9. Let the four soft subsets A_E, B_E, H_E and L_E of $\widetilde{X}$ be defined as follows:

$$A(e_1) = \{y, w\},\ A(e_2) = \{w, z\};$$
$$B(e_1) = \{y, z\},\ B(e_2) = \{x, y\};$$
$$H(e_1) = \{x, w\},\ H(e_2) = X; \text{ and}$$
$$L(e_1) = \{z\},\ L(e_2) = \{w, z\}.$$

Then we note the following:

(i) A_E and B_E are soft somewhere dense sets but their intersection is not soft somewhere dense.

(ii) H_E and L_E are soft *cs*-dense sets but their union is not soft *cs*-dense.

Proposition 2.23. *Let $\{M_{k_E} : k \in K\}$ be a class of soft subsets of (X, θ, E). Then $\widetilde{\bigcup}_{k\in K} M_{k_E}$ is soft somewhere dense if and only if $\widetilde{\bigcup}_{k\in K} cl(M_{k_E})$ is soft somewhere dense.*

Proof. Since $cl(\widetilde{\bigcup}_{k\in K} cl(M_{k_E})) = cl(\widetilde{\bigcup}_{k\in K} M_{k_E})$, then the proposition holds. □

Lemma 2.24. *If A_E is a soft open subset of (X, θ, E), then $A_E \widetilde{\bigcap} cl(B_E) \widetilde{\subseteq} cl$ $(A_E \widetilde{\bigcap} B_E)$ for each $B_E \widetilde{\subseteq} \widetilde{X}$.*

Proof. Let $P_e^x \in A_E \widetilde{\bigcap} cl(B_E)$. Then $P_e^x \in A_E$ and $P_e^x \in cl(B_E)$. Therefore for each soft open set U_E containing P_e^x, we have $U_E \widetilde{\bigcap} B_E \neq \widetilde{\emptyset}$. Since $U_E \widetilde{\bigcap} A_E$ is a non-null soft open set and $P_e^x \in U_E \widetilde{\bigcap} A_E$, then $(U_E \widetilde{\bigcap} A_E) \widetilde{\bigcap} B_E \neq \widetilde{\emptyset}$. Now, $U_E \widetilde{\bigcap} (A_E \widetilde{\bigcap} B_E) \neq \widetilde{\emptyset}$ implies that $P_e^x \in cl(A_E \widetilde{\bigcap} B_E)$. Hence

$$A_E \widetilde{\bigcap} cl(B_E) \widetilde{\subseteq} cl(A_E \widetilde{\bigcap} B_E).$$

□

Theorem 2.25. *If A_E is soft open and B_E is soft somewhere dense subsets of a soft hyperconnected space (X, θ, E), then $A_E \widetilde{\bigcap} B_E$ is soft somewhere dense.*

Proof. Consider that B_E is a soft somewhere dense subset of (X, θ, E). Then there exists a non-null soft open set G_E such that $G_E \widetilde{\subseteq} cl(B_E)$. Therefore $A_E \widetilde{\bigcap} G_E \widetilde{\subseteq} A_E \widetilde{\bigcap} cl(B_E) \widetilde{\subseteq} cl(A_E \widetilde{\bigcap} B_E)$. Since (X, θ, E) is soft hyperconnected, then $A_E \widetilde{\bigcap} G_E \neq \widetilde{\emptyset}$. Thus $A_E \widetilde{\bigcap} B_E$ is soft somewhere dense. □

Corollary 2.26. *If M_E is soft closed and N_E is soft cs-dense subsets of a soft hyperconnected space (X, θ, E), then $M_E \widetilde{\bigcup} N_E$ is soft cs-dense.*

Definition 2.27. A soft topological space (X, θ, E) is called strongly soft hyperconnected provided that a soft subset of $\widetilde{X}$ is soft dense if and only if it is non-null soft open.

One can directly notice that every strongly soft hyperconnected space is soft hyperconnected, however the converse is not always true as shown by the example below.

Example 2.28. Let E be a set of parameters and $\theta = \{\widetilde{\emptyset}, G_E : G_E^c$ is finite$\}$ be a soft topology on the set of real numbers $\mathcal{R}$. Then $(\mathcal{R}, \theta, E)$ is a soft hyperconnected space, but it is not a strongly soft hyperconnected space.

Theorem 2.29. *Let M_E and N_E be soft subsets of a strongly soft hyperconnected space (X, θ, E). If $int(M_E) = int(N_E) = \widetilde{\emptyset}$, then $int(M_E \widetilde{\bigcup} N_E) = \widetilde{\emptyset}$.*

Proof. If M_E or N_E are null, then the proof is trivial.

Let M_E and N_E be non-null soft sets satisfy the given condition. Suppose, to the contrary, that $int(M_E \widetilde{\bigcup} N_E) \neq \widetilde{\emptyset}$. Then there is $P_e^x \in int(M_E \widetilde{\bigcup} N_E)$ and this implies that there is a soft open set G_E containing P_e^x such that G_E

is contained in $M_E \widetilde{\bigcup} N_E$. Since (X, θ, E) is strongly soft hyperconnected, then we get

$$cl(M_E) \bigcup cl(N_E) = \widetilde{X}. \tag{1}$$

If $cl(M_E)$ is soft dense, then M_E is non-null soft open. But this contradicts that $int(M_E) = \widetilde{\emptyset}$. Therefore $\widetilde{\emptyset} \widetilde{\subset} cl(M_E) \widetilde{\subset} \widetilde{X}$. Similarly, $\widetilde{\emptyset} \widetilde{\subset} cl(N_E) \widetilde{\subset} \widetilde{X}$. Thus $\widetilde{\emptyset} \widetilde{\subset} (cl(M_E))^c \widetilde{\subset} \widetilde{X}$ and $\widetilde{\emptyset} \widetilde{\subset} (cl(N_E))^c \widetilde{\subset} \widetilde{X}$. From (1), we obtain

$$(cl(M_E))^c \widetilde{\bigcap} (cl(N_E))^c = \widetilde{\emptyset}.$$

But $(cl(M_E))^c$ and $(cl(N_E))^c$ are disjoint non-null soft open sets and this contradicts that (X, θ, E) is strongly soft hyperconnected. As a contradiction arises by assuming that $int(M_E \widetilde{\bigcup} N_E) \neq \widetilde{\emptyset}$, then the theorem holds. □

We now investigate under what conditions the union of soft *cs*-dense sets is soft *cs*-dense.

Lemma 2.30. *If M_E is a soft cs-dense subset of a strongly soft hyperconnected space (X, θ, E), then $int(M_E) = \widetilde{\emptyset}$.*

Proof. Let M_E be a soft *cs*-dense subset of (X, θ, E). Then there is a soft closed set $F_E \neq \widetilde{X}$ containing $int(M_E)$. Suppose that $int(M_E) \neq \widetilde{\emptyset}$. Then $cl(F_E) = \widetilde{X}$ and this implies that a soft set F_E is soft open. But this contradicts that (X, θ, E) is strongly soft hyperconnected. Hence $int(M_E) = \widetilde{\emptyset}$. □

Theorem 2.31. *If M_E and N_E are soft cs-dense subsets of a strongly soft hyperconnected space (X, θ, E), then $M_E \widetilde{\bigcup} N_E$ is soft cs-dense.*

Proof. Consider that M_E and N_E are soft *cs*-dense subsets of (X, θ, E). Then there are two soft closed sets $F_E \neq \widetilde{X}$ and $H_E \neq \widetilde{X}$ such that $int(M_E) \widetilde{\subseteq} F_E$ and $int(N_E) \widetilde{\subseteq} H_E$. Since (X, θ, E) is strongly soft hyperconnected, then $int(M_E) = \widetilde{\emptyset}$, $int(N_E) = \widetilde{\emptyset}$ and $F_E \widetilde{\bigcup} H_E \neq \widetilde{X}$. From Theorem 2.29, we obtain

$$int(M_E) \widetilde{\bigcup} int(N_E) = int(M_E \widetilde{\bigcup} N_E) = \widetilde{\emptyset}.$$

Consequently, $int(M_E \widetilde{\bigcup} N_E) \widetilde{\subseteq} F_E \widetilde{\bigcup} H_E$. Hence the proof is completed. □

Corollary 2.32. *If M_E and N_E are soft somewhere dense subsets of a strongly soft hyperconnected space (X, θ, E), then $M_E \widetilde{\bigcap} N_E$ is soft somewhere dense.*

Proposition 2.33. *If (X, θ, E) is a strongly soft hyperconnected space, then $\tau = SS(\theta) \bigcup \{\widetilde{\emptyset}\}$ forms a soft topology on X.*

Proof. Obviously, $\widetilde{X} \in SS(\theta) \subset \tau$, and from the definition of τ, we find that $\widetilde{\emptyset} \in \tau$. From Theorem 2.19 and Corollary 2.32, we find that τ is closed under arbitrary soft union and finite soft intersection. Hence τ is a soft topology on X. □

Lemma 2.34. *Let M_{E_i} be soft subsets of (X_i, θ_i, E_i) for $i = 1, 2, \ldots, n$. Then* $\prod_{i=1}^{i=s} cl(M_{E_i}) = cl(\prod_{i=1}^{i=s} M_{E_i})$.

Proof. We prove the lemma for two soft sets and one can prove it similarly for n soft sets.
Necessity: Let $P^{(x,y)}_{(\alpha,\beta)} \in [cl(M_{E_1}) \times cl(M_{E_2})]$. Then $P^x_\alpha \in cl(M_{E_1})$ and $P^y_\beta \in cl(M_{E_2})$. Suppose, to the contrary, that $P^{(x,y)}_{(\alpha,\beta)} \notin [cl(M_{E_1} \times M_{E_2})]$. Then there exists a soft open set $H_{E_1\times E_2}$ containing $P^{(x,y)}_{(\alpha,\beta)}$ such that $H_{E_1\times E_2} \widetilde{\bigcap} (M_{E_1} \times M_{E_2}) = \widetilde{\emptyset}_{E_1\times E_2}$. This means that there exist soft open sets U_{E_1} and V_{E_2} such that $H_{E_1\times E_2} = U_{E_1} \times V_{E_2}$. In this case, we can note that $U_{E_1} \widetilde{\bigcap} M_{E_1} = \widetilde{\emptyset}_{E_1}$ and $V_{E_2} \widetilde{\bigcap} M_{E_2} = \widetilde{\emptyset}_{E_2}$. Thus $P^x_\alpha \notin cl(M_{E_1})$ and $P^y_\beta \notin cl(M_{E_2})$. This implies that $P^{(x,y)}_{(\alpha,\beta)} \notin [cl(M_{E_1}) \times cl(M_{E_2})]$. But this contradicts our assumption. Thus $P^{(x,y)}_{(\alpha,\beta)} \in [cl(M_{E_1} \times M_{E_2})]$. Hence $cl(M_{E_1}) \times cl(M_{E_2}) \widetilde{\subseteq} cl(M_{E_1} \times M_{E_2})$.
Sufficiency: Let $P^{(x,y)}_{(\alpha,\beta)} \in [cl(M_{E_1} \times M_{E_2})]$. Suppose, to the contrary, that $P^{(x,y)}_{(\alpha,\beta)} \notin [cl(M_{E_1}) \times cl(M_{E_2})]$. Then $P^x_\alpha \notin cl(M_{E_1})$ or $P^y_\beta \notin cl(M_{E_2})$. Say, $P^x_\alpha \notin cl(M_{E_1})$. Therefore there exists a soft open set U_{E_1} containing P^x_α such that $U_{E_1} \widetilde{\bigcap} M_{E_1} = \widetilde{\emptyset}_{E_1}$. Obviously, $U_{E_1} \times \widetilde{X_2}$ is a soft open set containing $P^{(x,y)}_{(\alpha,\beta)}$ satisfies that $(U_{E_1} \times \widetilde{X_2}) \widetilde{\bigcap} (M_{E_1} \times M_{E_2}) = \widetilde{\emptyset}_{E_1\times E_2}$. This means that $P^{(x,y)}_{(\alpha,\beta)} \notin [cl(M_{E_1} \times M_{E_2})]$. But this contradicts our assumption. Thus $P^{(x,y)}_{(\alpha,\beta)} \in [cl(M_{E_1}) \times cl(M_{E_2})]$. Hence $cl(M_{E_1} \times M_{E_2}) \widetilde{\subseteq} cl(M_{E_1}) \times cl(M_{E_2})$. This completes the proof. □

Theorem 2.35. *Let $(\prod_{i=1}^{i=s} X_i, T, E)$ be a finite product soft topological space. Then M_{E_i} is a soft somewhere dense subset of (X_i, θ_i, E_i), for each $i = 1, 2, \ldots, s$, if and only if $\prod_{i=1}^{i=s} M_{E_i}$ is a soft somewhere dense subset of $(\prod_{i=1}^{i=s} X_i, T, E)$.*

Proof. Necessity: Let M_{E_i} be a soft somewhere dense subset of (X_i, θ_i, E_i). Then there is a non-null soft open set G_{E_i} such that $G_{E_i} \widetilde{\subseteq} cl(M_{E_i})$. Therefore $G_{E_1} \times G_{E_2} \times \cdots \times G_{E_s} \widetilde{\subseteq} cl(M_{E_1}) \times cl(M_{E_2}) \times ... \times cl(M_{E_s}) = \prod_{i=1}^{i=s} cl(M_{E_i}) = cl(\prod_{i=1}^{i=s} M_{E_i})$. Thus $\prod_{i=1}^{i=s} M_{E_i}$ is a soft somewhere dense subset of $(\prod_{i=1}^{i=s} X_i, T, E)$.
Sufficiency: Let $\prod_{i=1}^{i=s} M_{E_i}$ be a soft somewhere dense subset of $(\prod_{i=1}^{i=s} X_i, T, E)$. Then there is a non-null soft open set $G_{E_1} \times G_{E_2} \times \cdots \times G_{E_s}$ of $(\prod_{i=1}^{i=s} X_i, T, E)$ such

that $G_{E_1} \times G_{E_2} \times \cdots \times G_{E_s} \subseteq cl(\prod_{i=1}^{i=s} M_{E_i})$. Therefore $G_{E_i} \subseteq cl(M_{E_i})$ for each $i = 1, 2, \ldots, s$. Thus M_{E_i} is a soft somewhere dense subset of (X_i, θ_i, E_i). $\square$

Corollary 2.36. *Let $(\prod_{i=1}^{i=s} X_i, T, E)$ be a finite product soft topological space. Then B_{E_i} is a soft cs-dense subset of (X_i, θ_i, E_i), for each $i = 1, 2, \ldots, s$ if and only if $\bigcup_{i=1}^{i=s}(B_{E_i} \times \prod_{j=1, j\neq i}^{j=s} X_j)$ is a soft cs-dense subset of $(\prod_{i=1}^{i=s} X_i, T, E)$.*

Definition 2.37. A soft map $f_\phi : (X, \theta, A) \to (Y, \tau, B)$ is called soft bi-continuous if it is soft continuous and soft open.

Theorem 2.38. *If a soft map $f_\phi : (X, \theta, A) \to (Y, \tau, B)$ is soft bi-continuous, then the image of each soft somewhere dense set is soft somewhere dense.*

Proof. Let D_A be a soft somewhere dense subset of (X, θ, A). Then there is a soft open set G_A such that $G_A \widetilde{\subseteq} cl(D_A)$. Now, $f_\phi(G_A) \widetilde{\subseteq} f_\phi(cl(D_A))$. Because f_ϕ is soft open and soft continuous, then $f_\phi(G_A)$ is soft open and $f_\phi(cl(D_A))$ $\widetilde{\subseteq} cl(f_\phi(D_A))$. Hence $f_\phi(D_A)$ is a soft somewhere dense subset of (Y, τ, B). $\square$

Corollary 2.39. *If $\prod_{i\in I} M_{E_i}$ is a soft somewhere dense subset of a product soft topological space $(\prod_{i\in I} X_i, T, E)$, then M_{E_i} is a soft somewhere dense subset of (X_i, θ_i, E_i), for each $i \in I$.*

Corollary 2.40. *The property of being a soft somewhere dense set is a soft topological property.*

Theorem 2.41. *If (X, θ, E) is strongly soft hyperconnected, then $SS(\theta)$ forms a soft filter on X.*

Proof. It can be seen the following properties for $SS(\theta)$:

(i) From the definition of soft somewhere dense sets, $\widetilde{\emptyset} \notin SS(\theta)$.

(ii) From Corollary 2.32, $A_E \widetilde{\bigcap} B_E \in S(\theta)$, for each $A_E, B_E \in SS(\theta)$.

(iii) If $A_E \in SS(\theta)$ and $A_E \widetilde{\subseteq} B_E$, then it can be inferred that $B_E \in SS(\theta)$.

Hence $SS(\theta)$ is a soft filter on X. $\square$

Theorem 2.42. *A collection of all soft cs-dense subsets of a strongly soft hyperconnected space (X, θ, E) forms a soft ideal on X.*

Proof. Let $\mathcal{I}$ be a collection of all soft cs-dense subsets of a strongly soft hyperconnected space (X, θ, E). Then we have the following properties for $\mathcal{I}$:

(i) Obviously, $\widetilde{X} \notin \mathcal{I}$.

(ii) From Theorem 2.31, $A_E \widetilde{\bigcup} B_E \in \mathcal{I}$, for each $A_E, B_E \in \mathcal{I}$.

(iii) Let $B_E \in \mathcal{I}$ and $A_E \widetilde{\subseteq} B_E$. Since B_E is a soft cs-dense set, then from Corollary 2.6, we obtain $A_E \in \mathcal{I}$.

Hence $\mathcal{I}$ is a soft ideal on X. □

Lemma 2.43. *There exists an infinite soft somewhere dense subset D_E of an infinite soft topological space (X, θ, E) such that D^c_E is infinite.*

Proof. If there exists a finite soft somewhere dense subset of (X, θ, E), then the proof is trivial. So let $\Lambda = \{D_{i_E} : i \in I\}$ be a family of all soft somewhere dense subsets of (X, θ, E) such that D_{i_E} is infinite, for each $i \in I$. Since $\widetilde{X} \in \Lambda$, then $I \neq \widetilde{\emptyset}$. Suppose that every $D^c_{i_E}$ is finite. Then every soft closed set is finite as well. Take two infinite disjoint soft subsets A_E and B_E of $\widetilde{X}$ such that $A_E \widetilde{\bigcup} B_E = \widetilde{X}$. So $cl(A_E) = cl(B_E) = \widetilde{X}$ and $int(A_E) = int(B_E) = \widetilde{\emptyset}$. This implies that the soft sets A_E and B_E are both soft somewhere dense and soft *cs*-dense. But this contradicts our assumption. Hence the lemma holds. □

Theorem 2.44. *Let $\Lambda = \{D_{i_E} : i \in I\}$ be a collection of all infinite soft somewhere dense subsets of an infinite soft topological space (X, θ, E) such that $D^c_{i_E}$ is infinite, for each $i \in I$. Then an index set I is infinite.*

Proof. Let the given conditions be satisfied. Since $\{D^c_{i_E} : i \in I\}$ is a collection of all infinite soft cs-dense sets, then a collection of all infinite soft closed sets is contained in $\{D^c_{i_E} : i \in I\}$. Let $D_{i_{0_E}} \in \Lambda$. Then we have the next two cases:

(i) Either $cl(D_{i_{0_E}}) = \widetilde{X}$. Then $\{D_{i_{0_E}} \widetilde{\bigcup} \{P^x_e\}$: for each $P^x_e \in D^c_{i_{0_E}}\}$ is a collection of infinite soft somewhere dense sets. So $\{D_{i_{0_E}} \widetilde{\bigcup} \{P^x_e\}$: for each $P^x_e \in D^c_{i_{0_E}}\} \widetilde{\subseteq} \Lambda$.

(ii) Or there exists $j \in I$ such that $cl(D_{i_{0_E}}) = D^c_{j_E}$. Then an infinite soft set $D^c_{j_E}$ is both soft somewhere dense and soft cs-dense. So $\{D^c_{j_E} \widetilde{\bigcup} \{P^x_e\}$: for each $P^x_e \in D_{j_E}\}$ is a collection of infinite soft somewhere dense sets. Thus $\{D^c_{j_E} \widetilde{\bigcup} \{P^x_e\}$: for each $P^x_e \in D_{j_E}\} \widetilde{\subseteq} \Lambda$.

Since $D_{i_{0_E}}$ and $D^c_{j_E}$ are infinite in the two cases above, then an index set I is infinite. □

Corollary 2.45. *There is an infinite soft somewhere dense subset D_E of an infinite soft topological space (X, θ, E) contains P^x_e such that D^c_E is infinite, for each $P^x_e \in \widetilde{X}$.*

From the three results above, we conclude that any cover of an infinite soft topological space consists of soft somewhere dense sets has not a finite subcover. This conclusion will play a significant role to study new types of soft compact spaces depending on a notion of soft somewhere dense sets.

Theorem 2.46. *Let (X, θ, E) be a soft topological space such that $\mid E \mid= n$ and there exists $e_i \in E$ such that a topological space (X, θ_{e_i}) is hyperconnected. If $F(e)$ is a somewhere dense subset of (X, θ_e) for each $e \in E$, then F_E is a soft somewhere dense subset of (X, θ, E).*

Proof. Let $F(e_1)$ be a somewhere dense subset of (X, θ_e). Then there exists a non-empty open set $G_{j1}(e_1)$ such that $G_{j1}(e_1) \subseteq cl(F(e_1))$. In a similar way, there exist non-empty open sets $G_{j2}(e_2)$, $G_{j3}(e_3), \dots, G_{jn}(e_n)$ such that $G_{j2}(e_2) \subseteq cl(F(e_2))$, $G_{j3}(e_3) \subseteq cl(F(e_3)), \dots, G_{jn}(e_n) \subseteq cl(F(e_n))$. Now, we define a soft subset H_E as follows:

$$H(e_1) = G_{j1}(e_1) \bigcap G_{j2}(e_1) \bigcap \cdots \bigcap G_{jn}(e_1) \subseteq G_{j1}(e_1) \subseteq cl(F(e_1)).$$

$$H(e_2) = G_{j1}(e_2) \bigcap G_{j2}(e_2) \bigcap \cdots \bigcap G_{jn}(e_2) \subseteq G_{j2}(e_2) \subseteq cl(F(e_2)).$$

$$\vdots$$

$$H(e_n) = G_{j1}(e_n) \bigcap G_{j2}(e_n) \bigcap \cdots \bigcap G_{jn}(e_n) \subseteq G_{jn}(e_n) \subseteq cl(F(e_n)).$$

Putting $H_E = \widetilde{\bigcap}_{i=1}^{i=n} G_{jn_E}$. Obviously, H_E is a soft open subset of (X, θ, E) such that $H_E \widetilde{\subseteq} (cl(F))_E$. By hypotheses, there exists a hyperconnected topological space (X, θ_{e_i}) and this implies that $H(e_i) \neq \emptyset$. So H_E is a non-null soft open set. From Proposition 1.11, we obtain $(cl(F))_E \widetilde{\subseteq} cl(F_E)$. This completes the proof. □

Theorem 2.47. *If (A_E, θ_{A_E}, E) is a soft open subspace of (X, θ, E) and $L_E \widetilde{\subseteq} \widetilde{X}$, then $A_E \widetilde{\bigcap} cl(L_E) = (cl(A_E \widetilde{\bigcap} L_E))_{A_E}$, where $(cl(A_E \widetilde{\bigcap} L_E))_{A_E}$ is the soft closure of $A_E \widetilde{\bigcap} L_E$ in the soft subspace (A_E, θ_{A_E}, E).*

Proof. Consider $P_e^x \notin A_E \widetilde{\bigcap} cl(L_E)$. Then we have the following two possible cases:

(i) Either $P_e^x \notin A_E$. So $P_e^x \notin (cl(A_E \widetilde{\bigcap} L_E))_{A_E}$.

(ii) Or $P_e^x \in A_E$ and $P_e^x \notin cl(L_E)$. This means that there is a soft open subset G_E of (X, θ, E) containing P_e^x such that $G_E \widetilde{\bigcap} L_E = \widetilde{\emptyset}$. Now, $P_e^x \in G_E \widetilde{\bigcap} A_E$ and A_E is soft open. This implies that $(G_E \widetilde{\bigcap} A_E) \widetilde{\bigcap} (L_E \widetilde{\bigcap} A_E) = \widetilde{\emptyset}$. Thus $P_e^x \notin (cl(A_E \widetilde{\bigcap} L_E))_{A_E}$.

Thus $(cl(A_E \widetilde{\bigcap} L_E))_{A_E} \widetilde{\subseteq} A_E \widetilde{\bigcap} cl(L_E)$.

On the other hand, let $P_e^x \notin (cl(A_E \widetilde{\bigcap} L_E))_{A_E}$. Then a soft subspace (A_E, θ_{A_E}, E) containing a soft open set H_E satisfies that

$$P_e^x \in H_E \text{ and } H_E \widetilde{\bigcap} (A_E \widetilde{\bigcap} L_E) = \widetilde{\emptyset}.$$

By hypotheses, A_E is a soft open subset of $\widetilde{X}$, So H_E is also a soft open subset of (X, θ, E). Now, $H_E \widetilde{\bigcap} (A_E \widetilde{\bigcap} L_E) = (H_E \widetilde{\bigcap} A_E) \widetilde{\bigcap} L_E = \widetilde{\emptyset}$. then $P_e^x \notin A_E \widetilde{\bigcap} cl(L_E)$. Thus $A_E \widetilde{\bigcap} cl(L_E) \widetilde{\subseteq} (cl(A_E \widetilde{\bigcap} L_E))_{A_E}$. Hence the proof is complete. □

Corollary 2.48. *Let A_E be a soft dense subset of (X, θ, E) and $H_E \widetilde{\subseteq} A_E$. If H_E is a soft somewhere dense subset of (X, θ, E), then H_E is a soft somewhere dense subset of (A_E, θ_{A_E}, E).*

Proof. Let H_E be a soft somewhere dense subset of (X, θ, E). Then there exists a non-null soft open set G_E such that $G_E \widetilde{\subseteq} cl(H_E)$. Now, there exists a soft open set $F_E \in \theta_{A_E}$ such that $F_E = G_E \widetilde{\bigcap} A_E \widetilde{\subseteq} cl(H_E) \widetilde{\bigcap} A_E \widetilde{\subseteq} (cl(H_E))_{A_E}$. Since A_E is soft dense, then $F_E \neq \widetilde{\emptyset}$. Thus H_E is a soft somewhere dense subset of (A_E, θ_{A_E}, E). □

Conclusion

In 1999, Molodtsov [13] gave the novel idea of soft sets as a mathematical tool to copy with uncertainties and in 2011, Shabir and Naz [17] employed this notion to initiate the concept of soft topological spaces. The first appearance of generalized soft open sets was in 2013, by Chen [8]. The main propose of this work, is to present a soft somewhere dense sets notion as a new class of generalized soft open sets and study its main properties in detail. We first illustrate its relationships with the most famous generalized soft open sets, and verify that it contains all of them except for the null soft set. Under a condition of strongly soft hyperconnectedness, we point out that the union of soft *cs*-dense sets and the intersection of soft somewhere dense sets are soft *cs*-dense and soft somewhere dense, respectively. We also derive that a collection of soft somewhere dense subsets of a strongly soft hyperconnected space with a null soft set forms a soft topology on the universe set. We verify some significant results such as every soft set is soft somewhere dense or soft *cs*-dense, and the finite product of soft somewhere dense sets is soft somewhere dense. Moreover, we show that a number of infinite soft somewhere dense subsets of any infinite soft topological space is infinite. This result will be fundamental for studying soft compact spaces by utilizing a soft somewhere dense cover. Finally, we present some results which associate soft somewhere dense sets with some soft topological concepts such as soft topological properties and soft subspaces. In an upcoming paper, we plan to use an idea of soft somewhere dense sets to introduce new types of soft maps and soft separation axioms in soft topological spaces, and we make attempt to improve the significant application of regular open sets in digital topology which introduced in [4] by utilizing soft regular open sets.

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